



Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

1939-06

The effect of flexible mountings upon the resonant speeds of machines having unbalanced rotors.

MacKay, Hugh Trent

University of California

<http://hdl.handle.net/10945/6380>

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

THE EFFECT OF FLEXIBLE MOUNTINGS
UPON THE RESONANT SPEEDS OF
MACHINES HAVING UNBALANCED
MOTORS

Hugh Trent Mackay

The Effect of Flexible Mountings upon the Resonant Speeds
of Machines Having Unbalanced Rotors

By

Hugh Trent MacKay
B.S. (United States Naval Academy) 1930

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA

Approved:

.....
C.F.Garland

.....
L.M.K.Boelter

.....
R.E.Davis
Committee in Charge

Deposited in the University Library.....
Date June 1939 Librarian

The Effect of Viscosity on the Rate of
of Polymerization of Styrene

Thesis

1942

Submitted to the Faculty of the
University of California at Berkeley

THESIS

Submitted in partial fulfillment of the requirements for the degree of

Master of Science

in

Chemical Engineering

by

DAVID W. BROWN

of the

University of California

Approved:

.....
Prof. J. H. Duerksen

.....
Prof. J. H. Duerksen

.....
Prof. J. H. Duerksen

708.1

.....
Library of the University of California

TABLE OF CONTENTS

Title Page	Page 1
Table Of Contents	2
Introduction	3
Conclusions	6
Description Of Apparatus	7
Laboratory Method	11
Derivation Of Equations	15
Discussion	20
Sample Calculations	23
Curve Sheets	25
Data	33
References	38

INTRODUCTION

The subject of this paper was suggested by Assistant Professor C. F. Garland of the Mechanical Engineering Department of the University of California. It is an outgrowth of a graduate course in the dynamics of machinery presented at the University.

Any machine having a rotor is usually considered to be, in some degree, unbalanced. Therefore there will be a speed (or speeds) at which resonance will occur due to the frequency of the periodic force of the unbalanced rotor coinciding with the natural frequencies of the system concerned. Such speeds are termed resonant or critical speeds. The mountings of most machines are flexible in some measure. Even if the immediate foundation be considered absolutely rigid either the building or the ground upon which it rests may possess flexibility. In most cases this flexibility is not sufficient to appreciably affect the critical speed of the machine. There are cases, however, in which the mountings are relatively quite flexible for instance a diesel engine placed in a submarine.

It is possible that the flexible mounting would be a source of trouble. But it is also both conceivable and possible that the resulting phenomenon be put to advantageous use. Thus in one case a turbine was found to be operating very near its critical speed. The critical speed was then changed to a value removed from the proximity of the operating speed by making the supporting structure more elastic.

INTRODUCTION

The subject of this paper was suggested by a letter from
Dr. H. V. Johnston of the National Bureau of Standards
at the University of California. It is an attempt to show
that there is no question of a constant frequency of the
velocity.

Any motion having a velocity is usually considered to be

in some degree, unbalanced. Therefore there will be a force
(or forces) at which the motion will occur due to the force
of the velocity force of the motion. This force is
the natural frequency of the system concerned. Such a
system is present in optical systems. The motion of the
medium is periodic in some manner. Even if the motion
is considered as unbalanced, it is still within the limits
of the system upon which it acts and hence is periodic.

In each case the frequency is not considered as unbalanced
effect the optical speed of the motion. That the same, but
even, is what the motion is relative to the medium.
Therefore a small motion is a constant.

It is possible that the frequency motion is a
motion of the medium. But it is also possible that the
medium is moving. The motion is not in equilibrium. The
in one case a motion is found to be constant very near the
optical speed. The optical speed is found to be a
constant from the velocity of the motion. The motion is
constant relative to the medium.

Stodola (reference 3) mentions the possibility of the resonance of a rotor and its foundation. He concludes that the "sympathetic vibrations" will not lead to violent vibrations. Hort (reference 5) considers the problem in relation to the transfer of the machine vibrations to the foundations. He shows that at resonant speeds a large amount of energy is absorbed by the vibrating foundation. Rodgers (reference 6) gives equations for the amplitude of vibration of such a system in a general discussion of factors affecting critical speeds. Timoshenko (reference 2) considers the critical speeds of a machine somewhat similar to that taken up in this article.

In conducting the investigation for this paper amplitudes of vibration of an unbalanced rotor on a flexible mounting were measured at various speeds. Then the amplitudes of vibration were again measured over the same range of speeds with the mounting made rigid. Curves of the amplitude versus speed were plotted. This procedure was repeated for various values of the mass of the rotor and of the elastic constant of the mounting. At certain well defined speeds the amplitudes were found to be quite large. These particular speeds are the "resonant speeds" on which this study is focused.

Theoretical equations were then derived for the amplitude of vibration of the rotor by considering the apparatus as a coupled two body system with two degrees of freedom excited by the centrifugal force of the unbalanced rotor. The effect of friction was neglected. The expression for the resonant speeds was obtained from the equation for the amplitude of vibration. A dimensionless

22000 (reference 2) explains the possibility of the two-
 way of a rotor and the vibration, as compared with the "two-
 phase vibration" will not be a simple vibration. But
 (reference 2) considers the problem in relation to the rotation
 of the machine vibration to the foundation. In those cases it
 is assumed that a large amount of energy is absorbed by the
 rotor foundation. Rogers (reference 3) gives equations for the
 analysis of vibration of such a system in a general vibration
 of a rotor relative to the rotor. (reference 3)
 considers the critical speeds of a machine mounted on a
 base as in this article.

In conducting the investigation for this paper emphasis
 of vibration of an unbalanced rotor on a flexible foundation was
 assumed at various speeds. The two methods of vibration were
 again compared over the same range of speeds with the results
 that the curves of the amplitude versus speed were plotted.
 This procedure was repeated for various values of the mass of the
 rotor and of the elastic constant of the mounting. It was
 well noted that the amplitude was found to be quite large.
 These particular speeds are the "critical speeds" as used in
 many papers.

Theoretical equations were then derived for the analysis of
 vibration of the rotor by considering the system as a single
 two degree of freedom system of freedom defined by the rotor
 and the elastic constant of the mounting. The effect of the
 rotor on the foundation for the two cases was obtained
 from the analysis for the amplitude of vibration. A dimensionless

quantity was obtained by dividing these resonant speeds by the expression for the critical speed of the rotor when the mounting is rigid. Curves were plotted from the expression thus obtained using as a variable the ratio of the mass of the rotor to the mass of the part of the supporting structure that vibrates and using as a parameter the ratio of the elastic constant of the mounting to the elastic constant of the rotor. The derivations of all expressions used are presented under the heading "Derivation Of Equations".

The experimental and theoretical results are shown on curve sheets 1 and 2. The problem of this paper is considered in the various references as before mentioned. But in no reference that was consulted is there any consideration of a comparison of the experimental and theoretical results.

The flexibility of the mounting of the apparatus used for experimentation could be varied in the vertical plane only. Therefore this paper is limited to consideration of vibrations in the vertical plane.

CONCLUSIONS

(1) An unbalanced rotor on an elastic shaft carried by an elastic mounting has two resonant (critical) speeds. Both of these speeds are different from the one critical speed of the same rotor on a rigid mounting.

(2) Of the two critical speeds of a rotor with an elastic mounting one is above and one below the critical speed of the same rotor with a rigid mounting.

(3) The two critical speeds of a rotor with elastic mounting correspond to the two natural frequencies of an elastically coupled two mass system.

(4) For each of the two critical speeds the ratio of critical speed with flexible mounting to the critical speed with rigid mounting increases with increase of the ratio of the mass of the rotor to the mass of the base. (See curve sheets 1 and 2.)

(5) For each of the critical speeds the ratio of critical speed with flexible mounting to the critical speed with rigid mounting increases with increase in the ratio of the elastic constant of the flexible mounting to the elastic constant of the rotor. (See curve sheets 1 and 2.)

(6) The effect of damping on the numerical value of the critical speeds was very slight in the equipment used for experimentation.

There is a high possibility that the information provided in this document is not accurate and should not be used for any purpose.

(2) Of the few critical aspects of a water visit to a water

(1) The two witness reports of a person with black hair and a mustache on the two separate photographs of an individual furnished the same details.

1. The Board of Directors of the Corporation has approved the plan of reorganization and the plan of liquidation of the Corporation and the plan of distribution of the assets of the Corporation.

(b) The amount of funding we are requesting is \$100,000.

DESCRIPTION OF APPARATUS

The following description refers to figures 1 and 2.

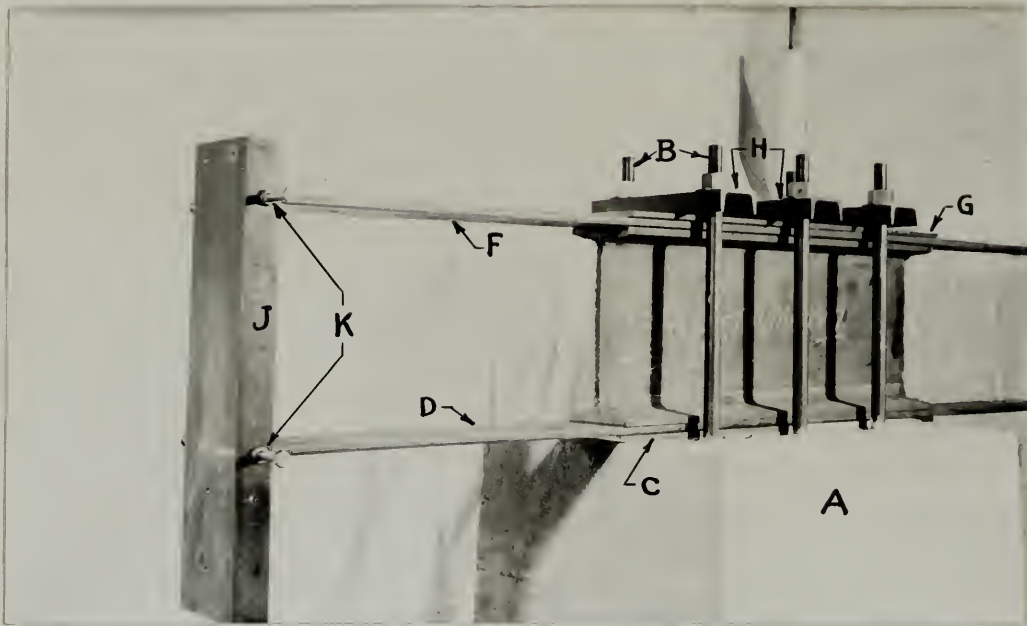
In assembling the apparatus a concrete foundation A, 26 inches high, was cast. Imbedded in the foundation and extending 18 inches above it are six 1-inch bolts B. Between the rows of bolts a $3/8$ -inch steel plate was placed and leveled. Two $1/2$ -inch steel bars D and F, 5 inches wide and 72 inches long, and spaced by a 30-inch length of 12-inch I beam were clamped to the foundation by means of the channel sections H, the plate G and the bolts B. Thus the bars D and F act as cantilever springs and supply flexibility to the mounting. The length of the springs can be varied by sliding them relative to the foundation after releasing the pressure on them.

A vertical member J is attached to the free ends of the springs D and F by means of conical pivots K. The base M is secured to the top of the vertical piece J and is braced by bars of $1/2$ -inch angle iron N. The base is made of two sections of 2-inch angle iron joined together with webs of $1/8$ -inch plate welded in place. A $1/2$ -inch slot extending the length of the base provides an anchorage for the bolts which hold the motor and bearing pedestals in place.

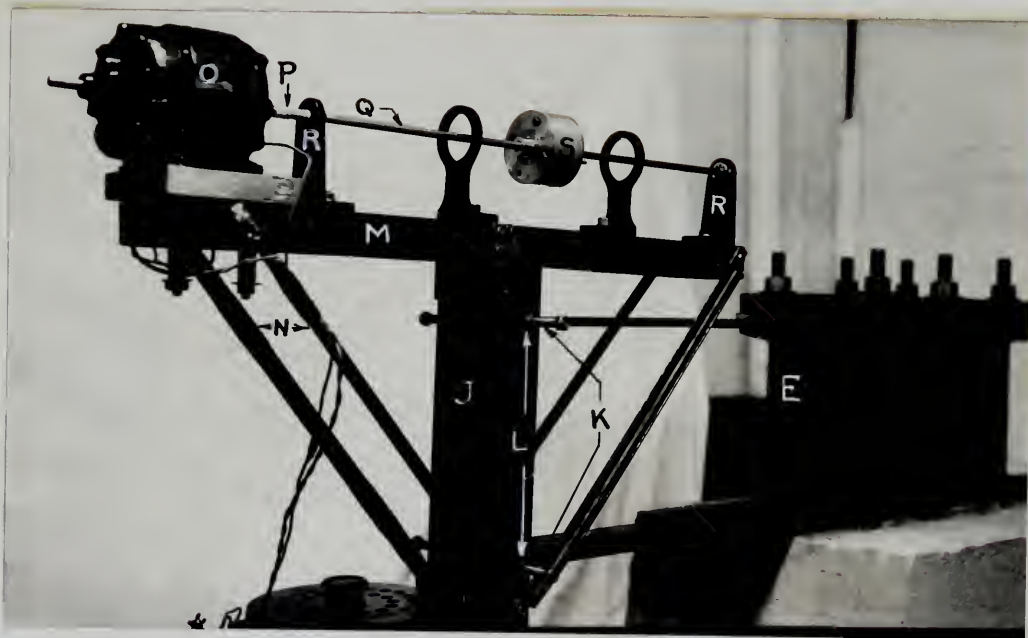
The $1/4$ -horsepower motor O is D.C. shunt wound and has a rated speed of 1750 r.p.m. The shaft of the motor is connected to the shaft carrying the rotor by a short section of $1/2$ -inch rubber tubing. This provides a satisfactory flexible coupling.

The shaft Q is of cold rolled steel, $7/16$ of an inch in

[illegible]



Elastic Mounting
Figure 1



Apparatus Completely Assembled
Figure 2

Classic bombing
Figure 1

Aggressive bombing
Figure 2

diameter. It is supported by two self-aligning ball bearings which are 28 $\frac{3}{8}$ inches between centers. The bearings are supported in pedestals R.

The rotor S mounted at the center of the shaft is made up of suitable end pieces and a variable number of annular brass discs. The end pieces are 4 inches in diameter and have four $\frac{1}{4}$ -inch holes equally spaced at a radius of $1\frac{1}{4}$ inches from the center. Set screws in the collars of the end pieces secure the rotor to the shaft. The brass discs of the rotor are 4 inches in diameter with a section 2 inches in diameter removed and with four $\frac{1}{4}$ -inch holes placed the same as in the end pieces. The discs and end pieces are held together by means of bolts through the $\frac{1}{4}$ -inch holes. Thus the several discs form a hollow rotor. This reduces the restraint on the bending of the shaft. In order to make the rotor unbalanced only three bolts are used to hold it together.

A 50-ohm rheostat is used in the armature circuit to control the speed of the motor.

A "Strobotac", manufactured by the General Radio Company, is used for measuring speeds of the rotor. A hand tachometer was used on preliminary runs. It was discarded when it was found that the speed of the rotor changed 20 or more r.p.m. when the tachometer was applied to the end of the shaft.

A dial guage is used to measure deflections of the base. It is placed at the free end of the lower cantilever spring D and held in position by a stand which is made fast to the floor.

Aluminum. It is supported by two self-aligning ball bearings
 and is 28 1/2 inches between centers. The bearings are
 located in members A.

The rotor is mounted at the center of the shaft in such
 as to align the shaft and a vertical member of member B
 above. The shaft is 2 inches in diameter and has four
 1-inch holes axially spaced at a center of 1 1/2 inches from the
 center. The shaft is the center of the shaft and the
 center is the shaft. The shaft is of the type of a shaft
 in diameter with a section 2 inches in diameter mounted and with
 four 1-inch holes spaced the same as in the shaft. The shaft
 and the shaft are the shaft in terms of shaft diameter and
 1-inch holes. The shaft is 2 inches from a shaft center. This
 member is mounted on the center of the shaft. It is in
 with the shaft and the shaft is 2 inches from the shaft. It is in

member.

A 50-mm diameter is used in the shaft and is in

the shaft of the shaft.

A "Sprocket", manufactured by the General Electric Company,

is used for mounting on the shaft. A shaft is mounted on
 the shaft and is 2 inches from the shaft. It is in the shaft
 the shaft of the shaft and is 2 inches from the shaft.
 shaft is applied to the end of the shaft.

A shaft is used to transmit motion of the shaft. It

is placed at the end of the shaft and is 2 inches from the shaft.

It is mounted by a shaft and is 2 inches from the shaft.

When it is desired to secure the mounting rigidly a section of 4-inch channel beam bent at right angles is bolted to the bottom of the vertical piece J and to a fitting in the floor. This gives a practically non-flexible mounting. No motion of the springs is discernible. There is, however, very slight flexibility in the base M.

It is difficult to believe that a person
 of such small size could be so strong. This is
 at the very least a case of a person in the
 a very/only very/only very/only very/only
 a very/only very/only very/only very/only
 a very/only very/only very/only very/only

6.

LABORATORY PROCEDURE

The elastic constant of the mounting was varied by changing the length of the flat bars that serve as springs. The mass of the rotor was varied by changing the number of brass discs used.

For each change of either spring constant of the mounting or of mass of the rotor the following procedure was carried out. At various speeds from 100 to 1500 r.p.m. the amplitudes of vibration of the mounting and of the rotor were measured and recorded. Each of these runs was made with both increasing and decreasing values of speed. The intervals of speed at which readings were made depended on the proximity of the critical speed. At the critical speeds several checks of the speed and amplitude were made to insure greater accuracy.

Then with the base made rigid the speeds and amplitudes of vibration were measured and recorded with both increasing and decreasing values of speed.

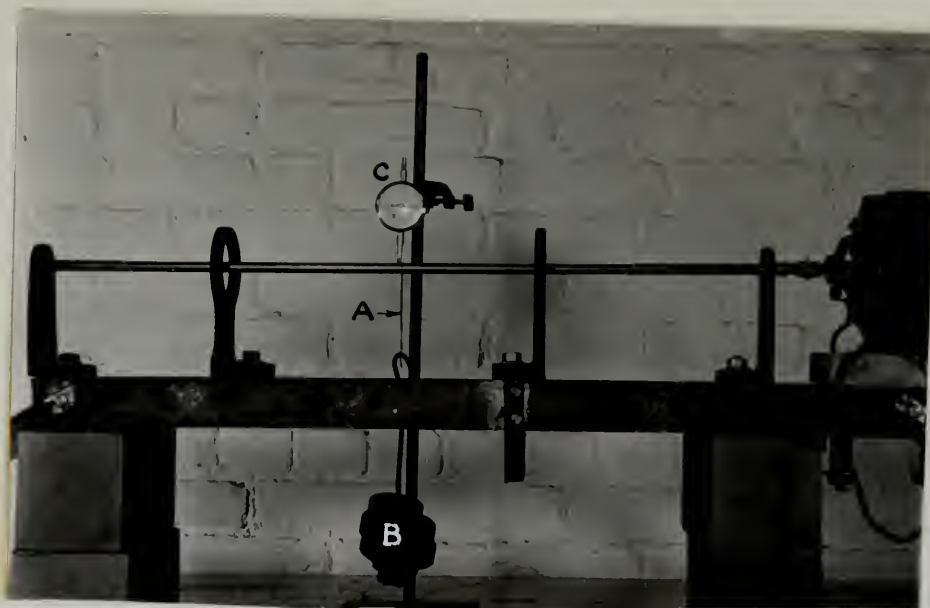
A cathetometer was used to measure the amplitude of the rotor. The instrument used had a least count of .0001 foot. Before starting a run a zero position was fixed by sighting the cathetometer at the topmost point on the rotor and then slowly rotating the rotor to check its position. Then at any desired speed the highest point of the rotor during vibration was sighted with the cathetometer. The difference between the two readings was recorded as the amplitude of vibration.

In obtaining the deflections of the base a dial guage with a least count of .001 inch was placed at the outer end of the lower spring. Thus at any speed the total movement of the base was read on the indicator.

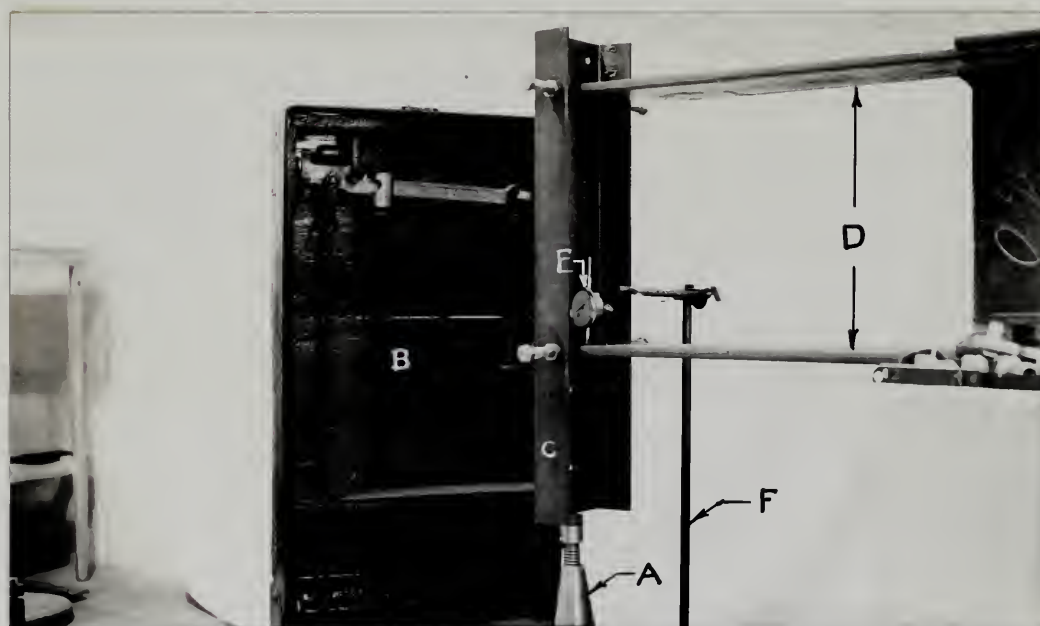
Speeds of revolution of the rotor were measured with a "Strobotac". This instrument was calibrated and kept regulated to read correctly.

The method used to determine the elastic constant of the shaft will be explained with the aid of figure 3. A loop of fine wire A was hung from the middle of the shaft. Various known weights were attached to the wire at B. Then the dial guage C placed at the middle of the shaft indicated the deflections caused by the weights. From the values obtained a curve of deflection versus applied weight was plotted. Then from this plot the elastic constant of the shaft was readily computed. Points on the curve were checked by rotating the shaft to different positions for each change of the weights. (See curve sheet 3.)

The elastic constant of the mounting was determined by a somewhat similar method. Figure 4 shows the apparatus used. A screw jack A was placed on the platform of the scales B. The top of the jack was made to exert force on the bottom of the vertical piece C which is attached to the cantilever springs D. Then as force was increased by extending the jack it was measured by the scales. The deflection of the springs was indicated by the dial guage E. This guage was held on stand F which was secured to the floor. Values obtained by this method were checked by deflecting the springs in the opposite direction with known weights placed on top of vertical piece C. Then a curve was plotted with deflection versus applied force



Apparatus For Determining The Elastic Constant Of The Shaft
Figure 3



Apparatus For Determining The Elastic Constant Of The Mounting
Figure 4



Figure 3
Schematic diagram of the experimental setup for the investigation of the effects of the size of the particles on the rate of the reaction.



Figure 4
Schematic diagram of the experimental setup for the investigation of the effects of the size of the particles on the rate of the reaction.

as coordinates. From this curve the elastic constant of the mounting was computed. This procedure was repeated for each length of cantilever springs used. (See curve sheet 4).

as described. From this curve the kinetic constant of the
reaction was computed. This constant was compared with the
value of 0.0011 per second reported by the author.

DERIVATION OF EQUATIONS

In making the derivation of the theoretical equations for the natural frequencies of the system under consideration the following conditions were considered to hold:

(1) The shaft and the cantilever springs are of uniform cross-section and homogeneous material.

(2) The shaft and the cantilever springs obey Hooke's law throughout the range over which they are deflected.

(3) In computing frequencies the effect of the distributed mass of the shaft may be accounted for by considering 17/35 of its amount to be concentrated at its middle with the rotor. This assumes a static deflection curve. (Reference 2, page 85.)

(4) In computing frequencies the effect of the distributed mass of the cantilever springs may be accounted for by considering 33/140 of their amount to be concentrated at their free end. This assumes a static deflection curve. (Reference 2, page 86.)

(5) Steady state motion prevails for all considerations taken up in this paper.

(6) Damping is negligible

Considering the system to be compound with two degrees of freedom it may be represented diagrammatically as in figure 5.

The following symbols, subscripts, and units are used:

m_1 - mass of rotor including proper portion of shaft--
lbs. sec²/in.

k_1 - elastic constant of shaft--lbs./in.

y_1 - displacement of the rotor--inches

APPENDIX A

in writing the description of the proposed conditions for the proposed system of the proposed conditions the following conditions were considered to be:

(1) The first and the second conditions are of the same nature and are considered to be:

- (2) The third and the fourth conditions are of the same nature and are considered to be:
- (3) In writing the description of the proposed conditions of the first and the second conditions the following conditions were considered to be:
- (4) In writing the description of the third and the fourth conditions the following conditions were considered to be:
- (5) In writing the description of the fifth and the sixth conditions the following conditions were considered to be:
- (6) In writing the description of the seventh and the eighth conditions the following conditions were considered to be:
- (7) In writing the description of the ninth and the tenth conditions the following conditions were considered to be:

and so on in this manner.

(8) The following conditions

concerning the system of the proposed conditions are of the same nature and are considered to be:

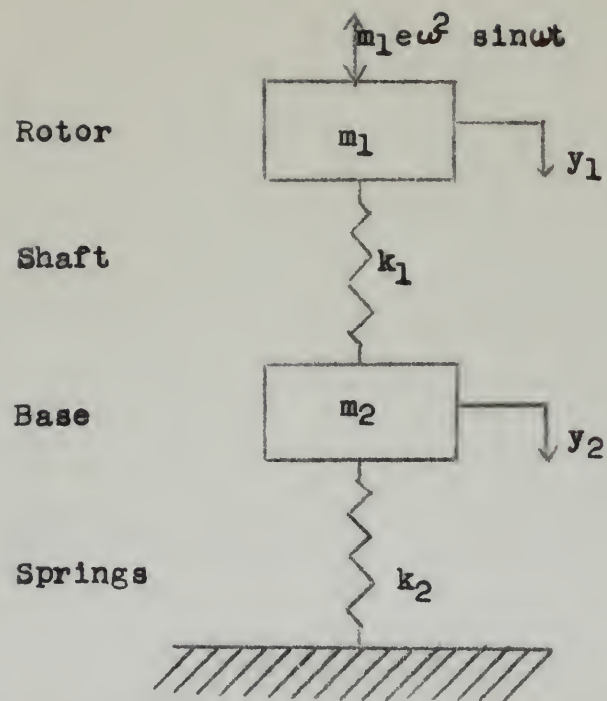
The following conditions, conditions, and so on are:

(9) The first condition is of the same nature and is considered to be:

(10) The second condition is of the same nature and is considered to be:

(11) The third condition is of the same nature and is considered to be:

(12) The fourth condition is of the same nature and is considered to be:



Diagrammatic Representation of the System
Used in Making the Mathematical Analysis

Figure 5

Figure 2
Used in Making the Mathematical Analysis
Kinematical Representation of the System



a_1 - amplitude of vibration of rotor--inches

m_2 - mass of base including proper portion of cantilever springs--lbs. sec²/in.

k_2 - elastic constant of the mounting--lbs./in.

y_2 - displacement of base--inches

a_2 - amplitude of vibration of base--inches

e - distance between geometrical center of rotor and its center of gravity--inches

ω - frequency of the exciting force--radians/sec.

$m_1 e \omega^2 \sin t$ - exciting force due to unbalanced rotor

Then considering the solution for the amplitudes of vibration for the steady state, the following force equations may be written for the two masses;

$$-k_1(y_1 - y_2) + m_1 e \omega^2 \sin \omega t = m_1 \ddot{y}_1 \quad (1)$$

$$k_1(y_1 - y_2) - k_2 y_2 = m_2 \ddot{y}_2 \quad (2)$$

Assume as solutions:

$$y_1 = a_1 \sin \omega t \quad y_2 = a_2 \sin \omega t$$

Then

$$\ddot{y}_1 = -a_1 \omega^2 \sin \omega t \quad \ddot{y}_2 = -a_2 \omega^2 \sin \omega t$$

Substituting these values in equations 1 and 2,

$$-k_1 a_1 \sin \omega t + k_1 a_2 \sin \omega t + m_1 e \omega^2 \sin t = -m_1 a_1 \omega^2 \sin t \quad (3)$$

$$k_1 a_1 \sin \omega t - k_1 a_2 \sin \omega t - k_2 a_2 \sin \omega t = -m_2 a_2 \omega^2 \sin \omega t \quad (4)$$

Let $\omega t = \pi/2$. Then $\sin \omega t = 1$. Substituting in equations 3 and 4,

$$-k_1 a_1 + k_1 a_2 + m_1 e \omega^2 = -m_1 a_1 \omega^2 \quad (5)$$

$$k_1 a_1 - k_1 a_2 - k_2 a_2 = -m_2 a_2 \omega^2 \quad (6)$$

Transposing,

$$a_1 (k_1 - m_1 \omega^2) - a_2 k_1 = m_1 e \omega^2 \quad (5)$$

$$-a_1 k_1 + a_2 (k_1 - k_2 - m_2 \omega^2) = 0 \quad (6)$$

Solving for a_1 by means of a determinant,

$$a_1 = \frac{\begin{vmatrix} m_1 e \omega^2 & -k_1 \\ 0 & (k_1 - k_2 - m_2 \omega^2) \end{vmatrix}}{\begin{vmatrix} (k_1 - m_1 \omega^2) & -k_1 \\ -k_1 & (k_1 - k_2 - m_2 \omega^2) \end{vmatrix}}$$

$$a_1 = \frac{m_1 e \omega^2 (k_1 - k_2 - m_2 \omega^2)}{(k_1 - m_1 \omega^2) (k_1 - k_2 - m_2 \omega^2) - k_1^2} \quad (7)$$

If the denominator of the last expression is equal to zero the value of a_1 becomes infinite. This corresponds to a resonant speed.

$$\text{Thus, } (k_1 - m_1 \omega^2) (k_1 - k_2 - m_2 \omega^2) - k_1^2 = 0$$

Expanding and dividing through by $m_1 m_2$,

$$\omega^4 \left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0$$

Solving by the quadratic equation,

$$\omega^2 = \frac{1}{2} \left[\left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1} \right) \pm \sqrt{\left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1} \right)^2 - \frac{4k_1 k_2}{m_1 m_2}} \right] \quad (8)$$

In order to obtain the dimensionless expression for ω/ω_0 expand, take the square root, and divide the right hand side

of equation 8 by $\sqrt{k/m}$. Where $\omega_c = \sqrt{k/m}$ which is the critical speed of the rotor when the mounting is rigid.

$$\frac{\omega}{\omega_c} = \sqrt{\frac{1}{2} \left[\frac{m_1 + \frac{k_2 m_1}{k_1 m_2} + 1}{\frac{m_2}{m_2} + \frac{k_2 m_1^2}{k_1 m_2^2} + \frac{2m_1}{m_2} - \frac{2k_2 m_1}{k_1 m_2} + \frac{k_2^2 m_1^2}{k_1^2 m_2^2} + 1 \right]}}$$

Then substitute $M = m_1 / m_2$ and $K = k_2 / k_1$,

$$\frac{\omega}{\omega_c} = \sqrt{\frac{1}{2} \left[(M + 1 + KM) \pm \sqrt{(M + 1)^2 + K(2M^2 - 2M) + K^2 M^2} \right]} \quad (9)$$

DISCUSSION

The agreement of the experimental values of resonant frequency with the computed values is very good as shown by curve sheets 1 and 2. The percentage difference of the experimentally found points from the computed curves varies from .5% to 4.5% with an average value of slightly less than 2%.

The near agreement of the points and the curves is considered as justification of the assumption that a rotor on an elastic shaft supported on an elastic mounting may be considered, in the ideal case, to be an elastically coupled two body system with two degrees of freedom in so far as vertical motion is concerned.

There are several reasons for the differences between the experimental and theoretical values of resonant frequencies. Personal and instrumental errors are always likely. It was, in some cases, difficult to measure the exact critical speed. This happened when the speed at which the amplitude of vibration of the rotor was unstable. In such instances the rotor would reach its greatest amplitude only momentarily and then go through the resonant condition and settle down to smooth running at a much higher speed. Another possible reason for the differences between the experimental and theoretical values is that friction and damping were present in various parts of the apparatus. The bearings were self-aligning but there was some friction in them. Also a small amount of pivot friction existed in the mounting.

There is a slight error (about 1%) produced in the formulas for critical speeds when portions of the shaft and of the cantilever springs are considered concentrated at points in order to correct for the distributed mass of these parts of the experimental apparatus. (See reference 2, page 86.)

The rotor tends to stiffen the shaft thereby reducing its effective length. Also the slight bearing restraint has the same effect. The flexibility of the bearing pedestals and of the base holding them tends to increase the effective length of the shaft. These two effects have apparently, at least partially, offset each other since by using the actual length of the shaft close agreement of experimental and theoretical results was obtained.

As noted in the derivation of equations damping was considered as having a negligible effect on the values of the critical speeds. However in the preliminary work the effect of damping was considered and investigated at some length. Den Hartog (reference 4, page 107) shows that relatively large values of the damping coefficient have little effect on the critical speeds. The magnitudes of the damping coefficients for the shaft and for the cantilever springs were obtained and found to be quite small. These values are tabulated on the data sheets.

In the analysis carried out in this paper vibrations in the vertical direction only are considered. However it should be noted that at various speeds the shaft was found to vibrate with the greatest amplitude in the horizontal plane. At still other

speeds the greatest amplitude of vibration was at an angle between the vertical and the horizontal. When constructing the laboratory apparatus it was made very rigid in the horizontal direction. Therefore the vibrations in the horizontal plane are very little affected by the flexibility of the mounting. Thus the former case above is the same as the critical speed if the mounting were rigid. The latter case represents a combination of the motion of the former case and the vertical motion existing when the base is free to vibrate. (See reference 2, page 212.)

Curve sheets 5 to 8 inclusive show that with the mounting rigid there is in each case a point in the neighborhood of 450 r.p.m. where the amplitude of the rotor increases. The actual increase is slight, varying from $1/100$ to $1/30$ of an inch, and hence would be of little practical importance. A possible explanation for this increase in amplitude is found in reference 3, article 19b in which the author, Stodola, discusses the effects of the weight of the rotor, variable elasticity of the shaft, and variable driving impulse. Any or all of these effects are applicable to the apparatus used for test. Moreover in the same article Stodola mentions a situation practically the same as that under consideration here for which he could find no reason for the increase in the amplitude of vibration.

SAMPLE CALCULATIONS

First, to obtain values for the elastic constant of the cantilever springs, the following values were read directly from curve sheet 4:

$$f_2 \text{ (force)} = 180 \text{ lbs.} \quad d_2 \text{ (deflection)} = .198 \text{ inch}$$

$$f_1 \text{ (force)} = \underline{10} \text{ lbs.} \quad d_1 \text{ (deflection)} = \underline{.007} \text{ inch}$$

$$\text{difference} = 170 \text{ lbs.} \quad \text{difference} = .191 \text{ inch}$$

$$\text{Therefore } k_2 = \frac{170}{.191} = 890 \text{ lbs. per inch}$$

In a similar manner the elastic constant for other lengths of the cantilever springs and for the shaft were computed.

Points for the theoretical curves of curve sheets 1 and 2 were obtained by substituting the experimental values of K and a series of arbitrary values of M into equation 9 which was derived in the "Derivation Of Equations".

$$\text{Thus } k_1 = 106.6 \quad k_2 = 890 \quad K = \frac{890}{106.6} = 8.39$$

Substituting this value of K and $M = .1$ into equation 9,

$$\begin{aligned} \omega/\omega_0 &= \sqrt{\frac{1}{4}(1.100 + .839) \pm \sqrt{1.210 + .872 + 1.678}} \\ &= 1.135 \text{ or } .808 \quad (\text{plotted on curve sheet 1}) \end{aligned}$$

The experimental data gave the following values from which an experimental point for curve sheet 1 is obtained.

$$m_1 = .0192 \quad m_2 = .205 \quad M = \frac{.0192}{.205} = .0934$$

$$\omega = 820 \text{ or } 565 \text{ (from curve sheet 5)}$$

$$\omega_0 = 740 \text{ (from curve sheet 5)}$$

$$\frac{\omega}{\omega_0} = 1.11 \text{ or } .765 \text{ (plotted on curve sheet 1)}$$

TABLE 1

These are values for the constant of the
 coefficient of the following series and also the

value of

$$\begin{aligned} C_1(1000) &= 1.00 \times 10^{-3} \\ C_2(1000) &= 1.00 \times 10^{-3} \\ C_3(1000) &= 1.00 \times 10^{-3} \\ C_4(1000) &= 1.00 \times 10^{-3} \\ C_5(1000) &= 1.00 \times 10^{-3} \\ C_6(1000) &= 1.00 \times 10^{-3} \\ C_7(1000) &= 1.00 \times 10^{-3} \\ C_8(1000) &= 1.00 \times 10^{-3} \\ C_9(1000) &= 1.00 \times 10^{-3} \\ C_{10}(1000) &= 1.00 \times 10^{-3} \end{aligned}$$

In a similar manner the value of the constant of the
 coefficient of the following series and also the

value of the constant of the following series and also the
 value of the constant of the following series and also the
 value of the constant of the following series and also the
 value of the constant of the following series and also the

$$C_1(1000) = 1.00 \times 10^{-3}$$

$$\begin{aligned} C_2(1000) &= 1.00 \times 10^{-3} \\ C_3(1000) &= 1.00 \times 10^{-3} \\ C_4(1000) &= 1.00 \times 10^{-3} \\ C_5(1000) &= 1.00 \times 10^{-3} \\ C_6(1000) &= 1.00 \times 10^{-3} \\ C_7(1000) &= 1.00 \times 10^{-3} \\ C_8(1000) &= 1.00 \times 10^{-3} \\ C_9(1000) &= 1.00 \times 10^{-3} \\ C_{10}(1000) &= 1.00 \times 10^{-3} \end{aligned}$$

The following series and also the value of the constant of the
 following series and also the value of the constant of the

$$C_1(1000) = 1.00 \times 10^{-3}$$

$$C_2(1000) = 1.00 \times 10^{-3}$$

$$C_3(1000) = 1.00 \times 10^{-3}$$

$$C_4(1000) = 1.00 \times 10^{-3}$$

The dimensionless coefficient of friction, γ , is given by the following expression:

$$\gamma = \frac{\log_e h_1/h_2}{\pi} = \frac{\log_e h_2/h_3}{\pi}$$

$$\gamma = \frac{2n}{\omega} = \frac{b}{\pi\omega}$$

b = coefficient of viscous damping force

h_1, h_2, h_3 = successive amplitudes of free vibration as shown on figure 6

Thus the value of γ for the rotor when $m_1 = .0116$ is computed below.

$$h_1 = .48 \text{ inch}$$

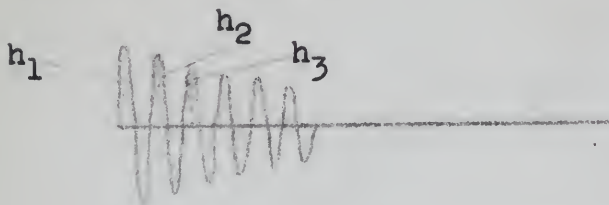
$$h_2 = .42 \text{ inch}$$

$$h_1/h_2 = 1.142$$

$$\log_e 1.142 = .1327$$

$$\gamma = .1327/3.1416 = .0422$$

Values of γ for the rotor with a different value of m_1 and for the cantilever springs at the various lengths were similarly computed.



Record of Free Vibration Made by Vibrograph

Figure 6

The following are the results of the analysis of the data given

$$Y = \frac{1}{2} \log \frac{1 + \sqrt{1 + 4X}}{1 - \sqrt{1 + 4X}}$$

$$K = \frac{2}{\pi} \log \frac{1 + \sqrt{1 + 4X}}{1 - \sqrt{1 + 4X}}$$

Y = constant of the reaction

It is a measure of the intensity of the reaction

and is given by

The value of Y for the reaction is 1.15

which is

$$Y = 1.15 \log \frac{1 + \sqrt{1 + 4X}}{1 - \sqrt{1 + 4X}}$$

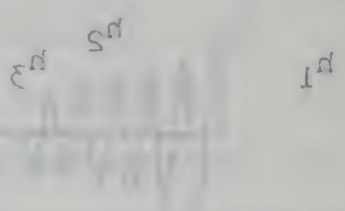
$$\log \frac{1 + \sqrt{1 + 4X}}{1 - \sqrt{1 + 4X}} = 1.15 Y$$

$$Y = 1.15 \log \frac{1 + \sqrt{1 + 4X}}{1 - \sqrt{1 + 4X}}$$

Values of Y for the reaction are 1.15

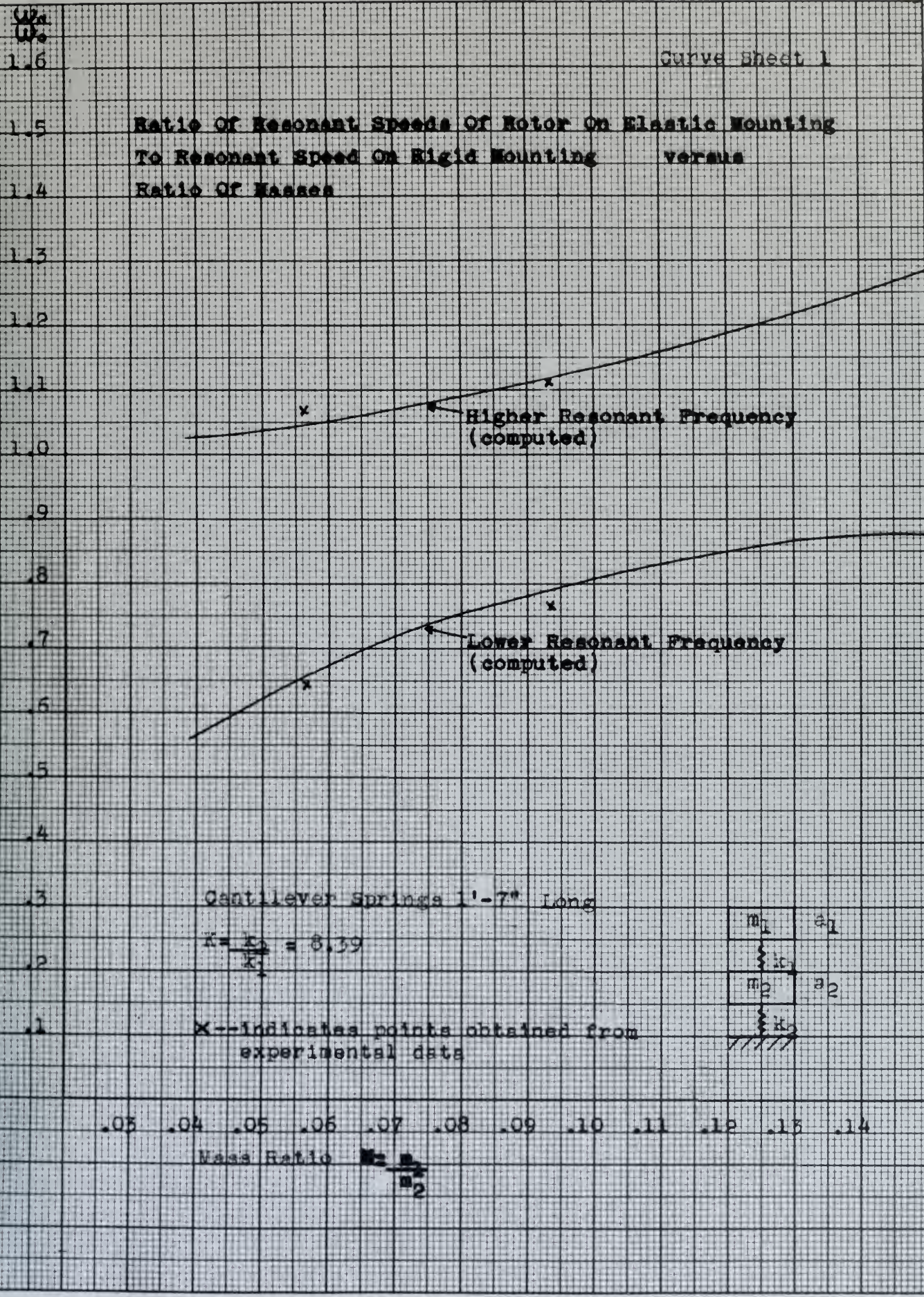
and for the reaction are 1.15

which is



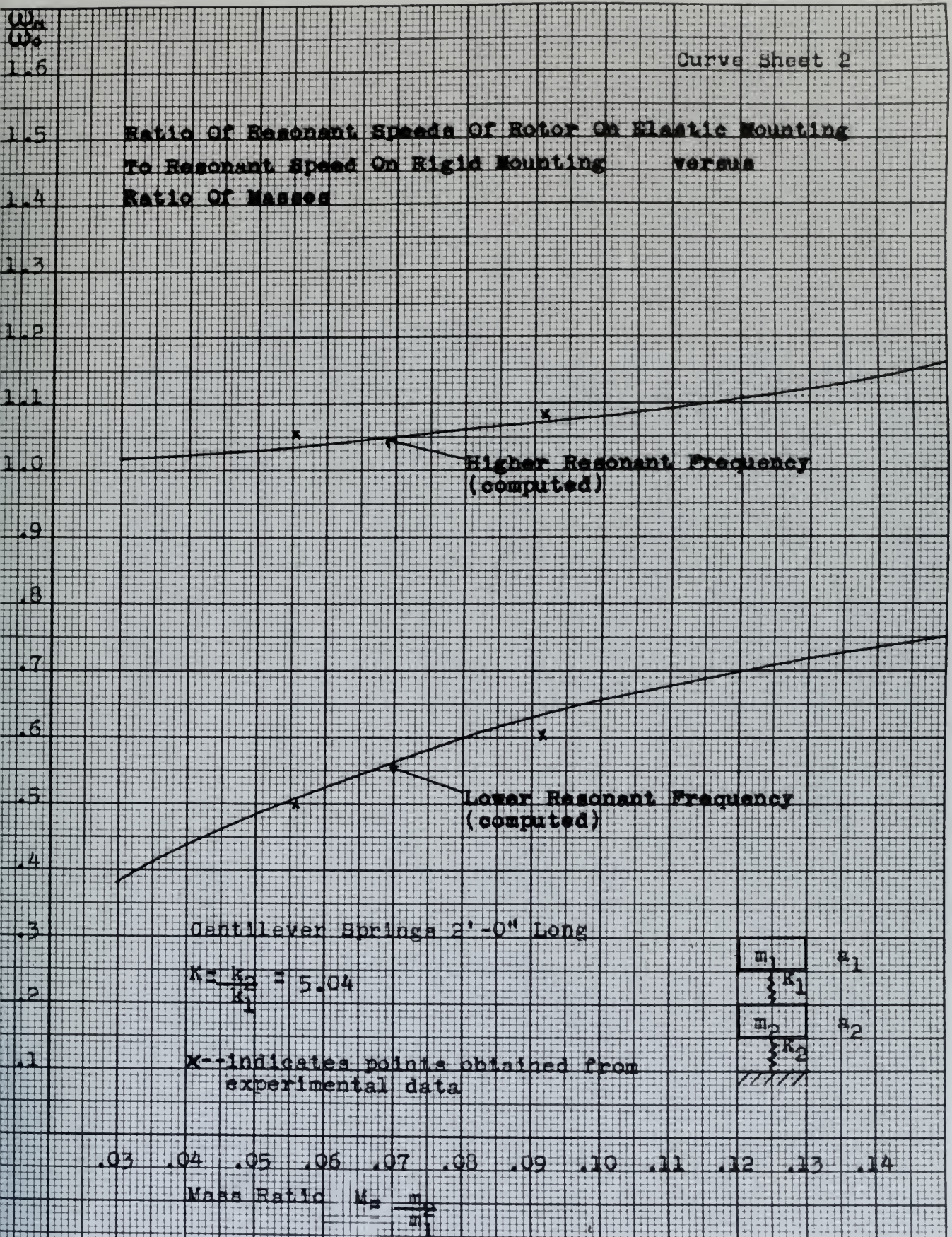
Values of Y for the reaction are 1.15

Figure 1



Curve Sheet 2

Ratio Of Resonant Speeds Of Rotor On Elastic Mounting
To Resonant Speed On Rigid Mounting versus
Ratio Of Masses



Curve Sheet 3

Deflection Of Shaft versus Applied Force

Force
(lbs.)

14

12

10

8

6

4

2

0

0

.04

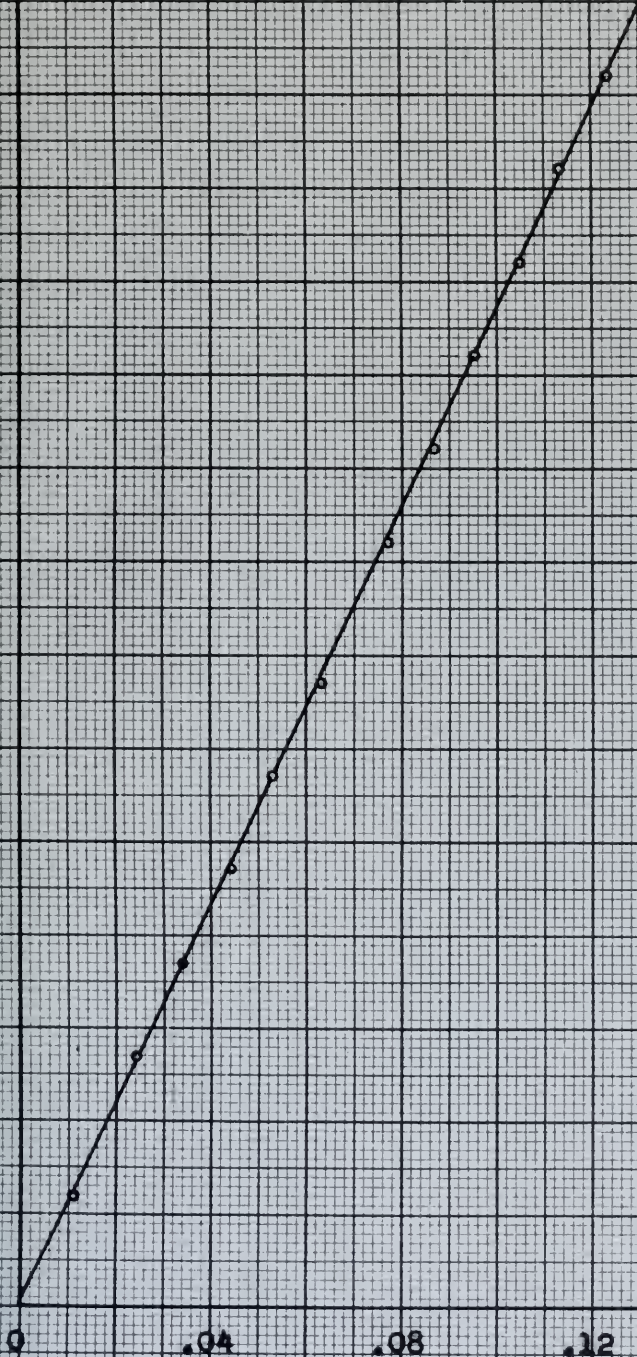
.08

.12

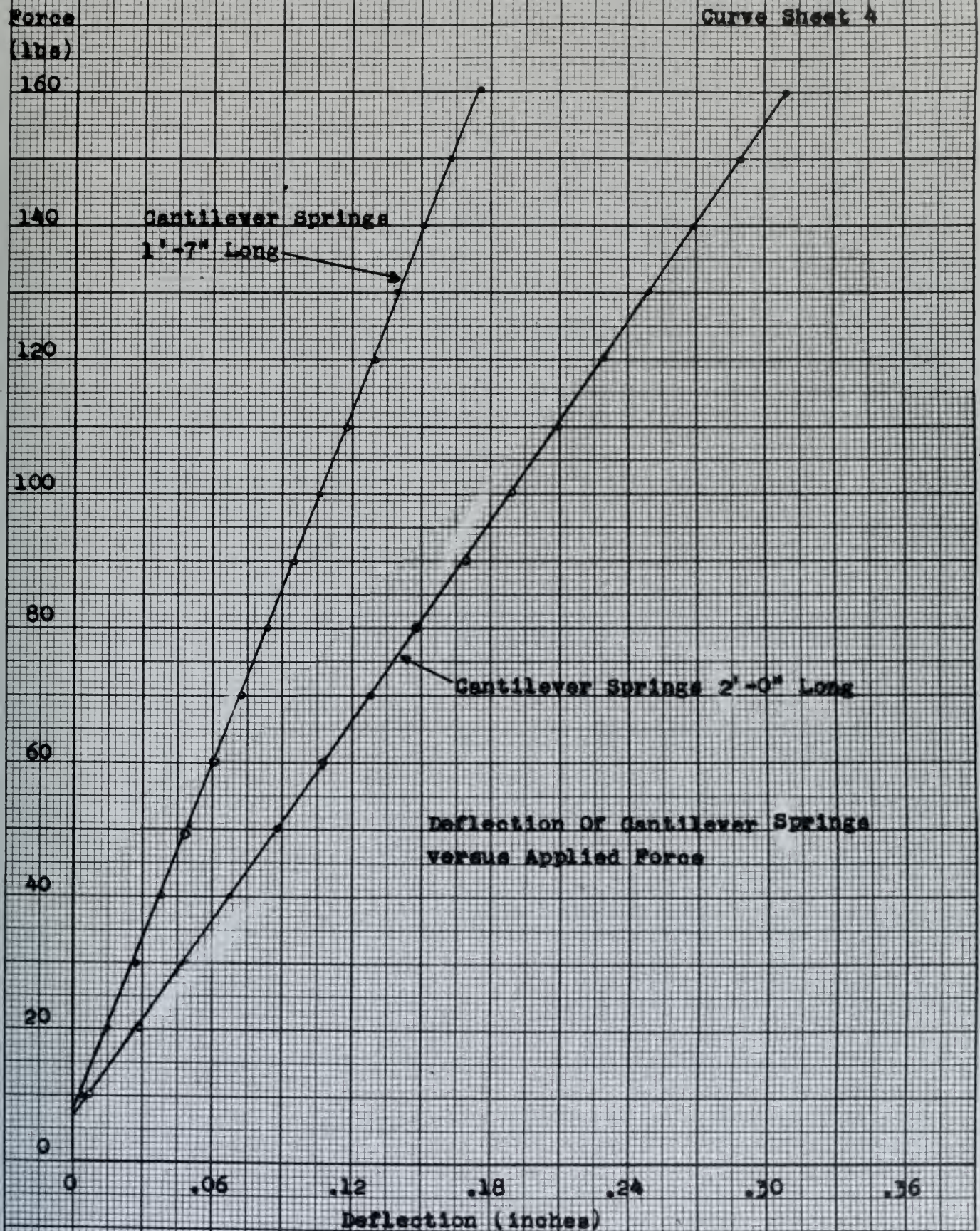
.16

.20

Deflection (inches)



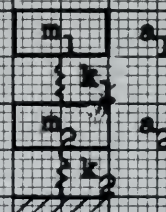
Curve Sheet 4



Amplitude a_1 (inches)

Amplitude Of Vibration Of Rotor versus Speed

Cantilever Springs 1'-7" Long
 $m_1 = .0192$ (8 discs)



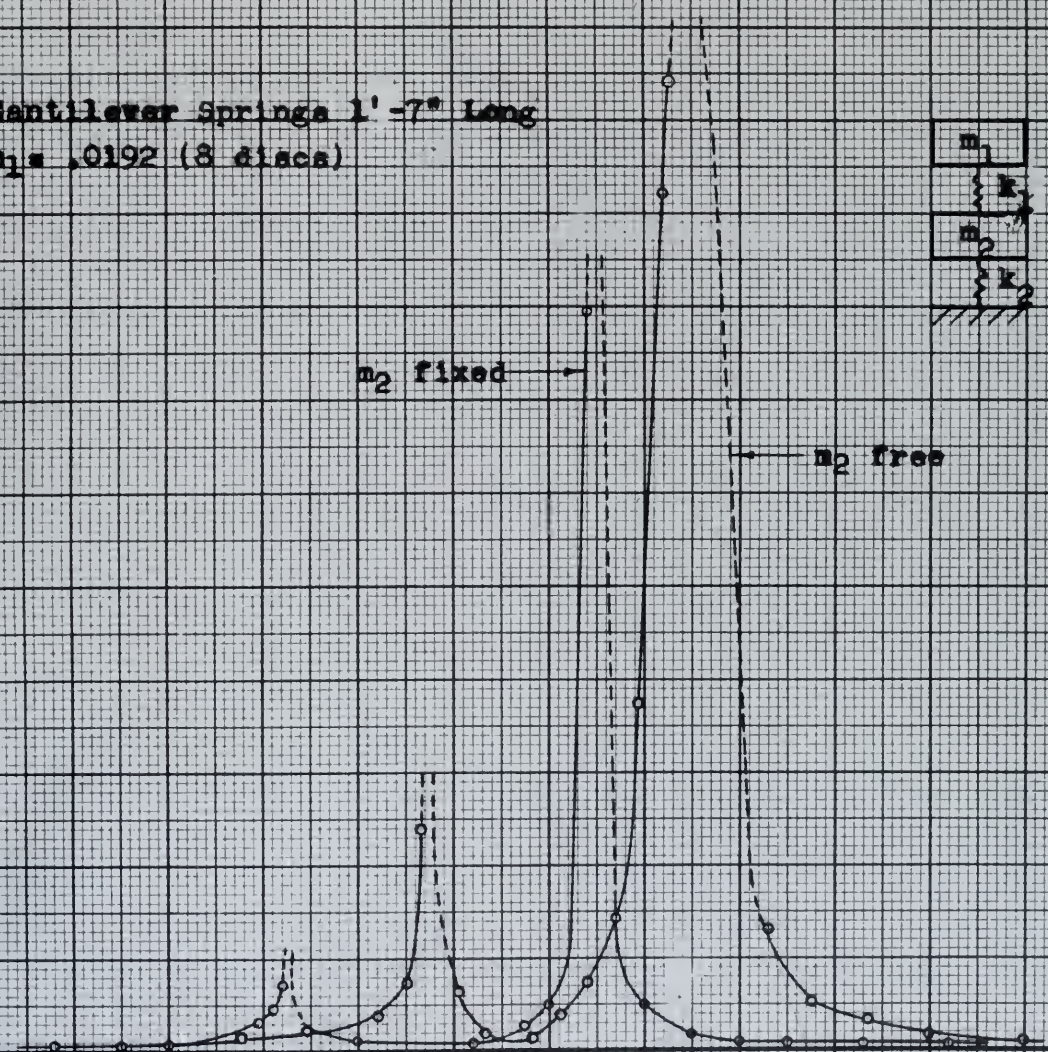
m_2 fixed

m_2 free

.60
 .50
 .40
 .30
 .20
 .10
 0

0 200 400 600 800 1000 1200

Revolutions Per Minute



Curve Sheet 6

Amplitude- a_1 -(inches)

.60

Amplitude Of Vibration Of Rotor Versus Speed

.50

Cantilever Springs 1'-7" Long

 $m_1 = .0116$ (4 discs)

.40

.30

 m_2 fixed

.20

 m_2 free

.10

0

0

200

400

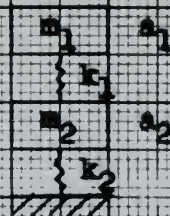
600

800

1000

1200

Revolutions Per Minute

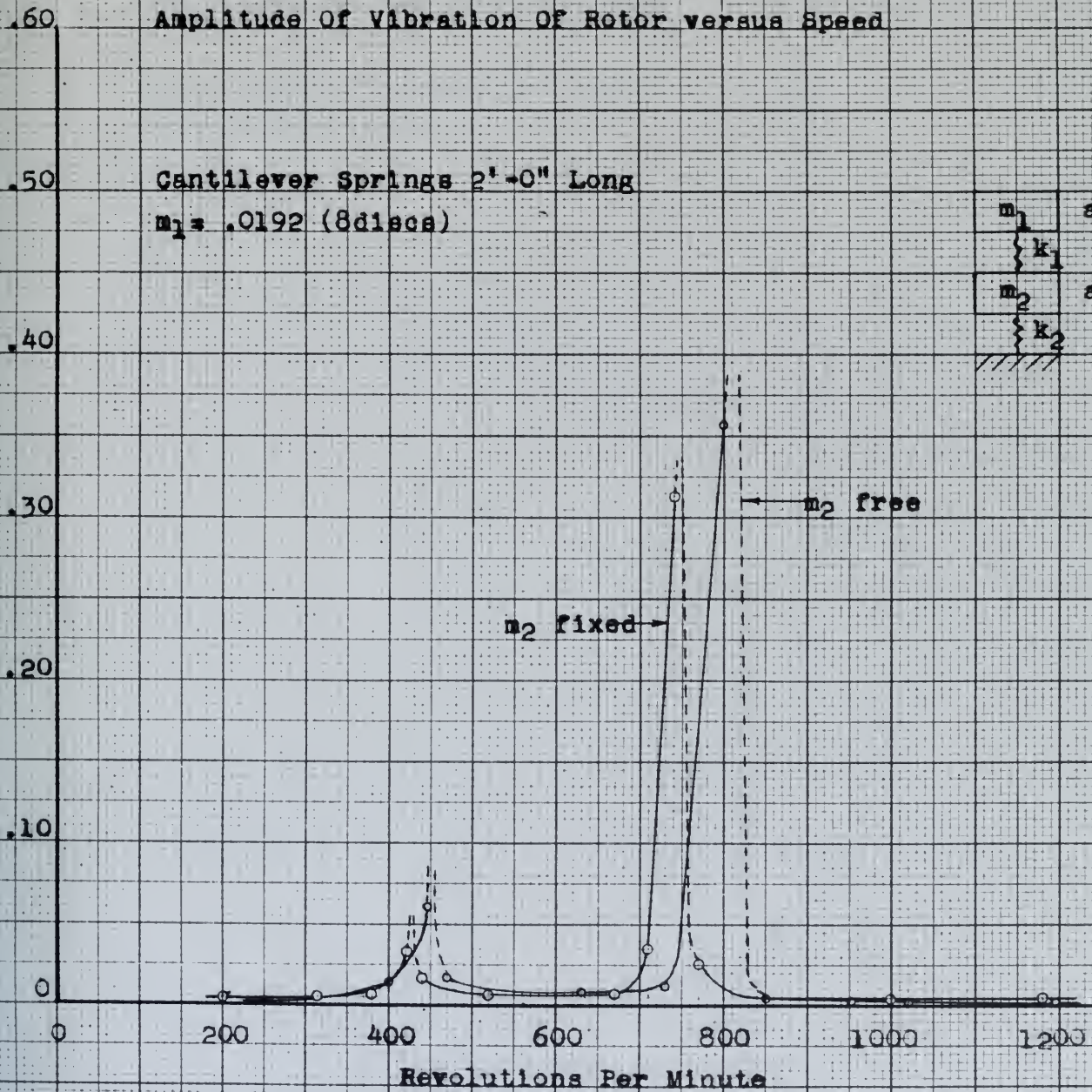
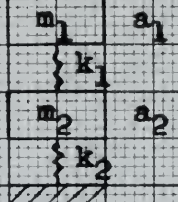


Curve Sheet 7

Amplitude a_1 (inches)

Amplitude Of Vibration Of Rotor versus Speed

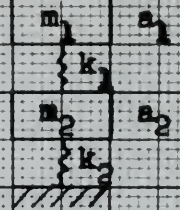
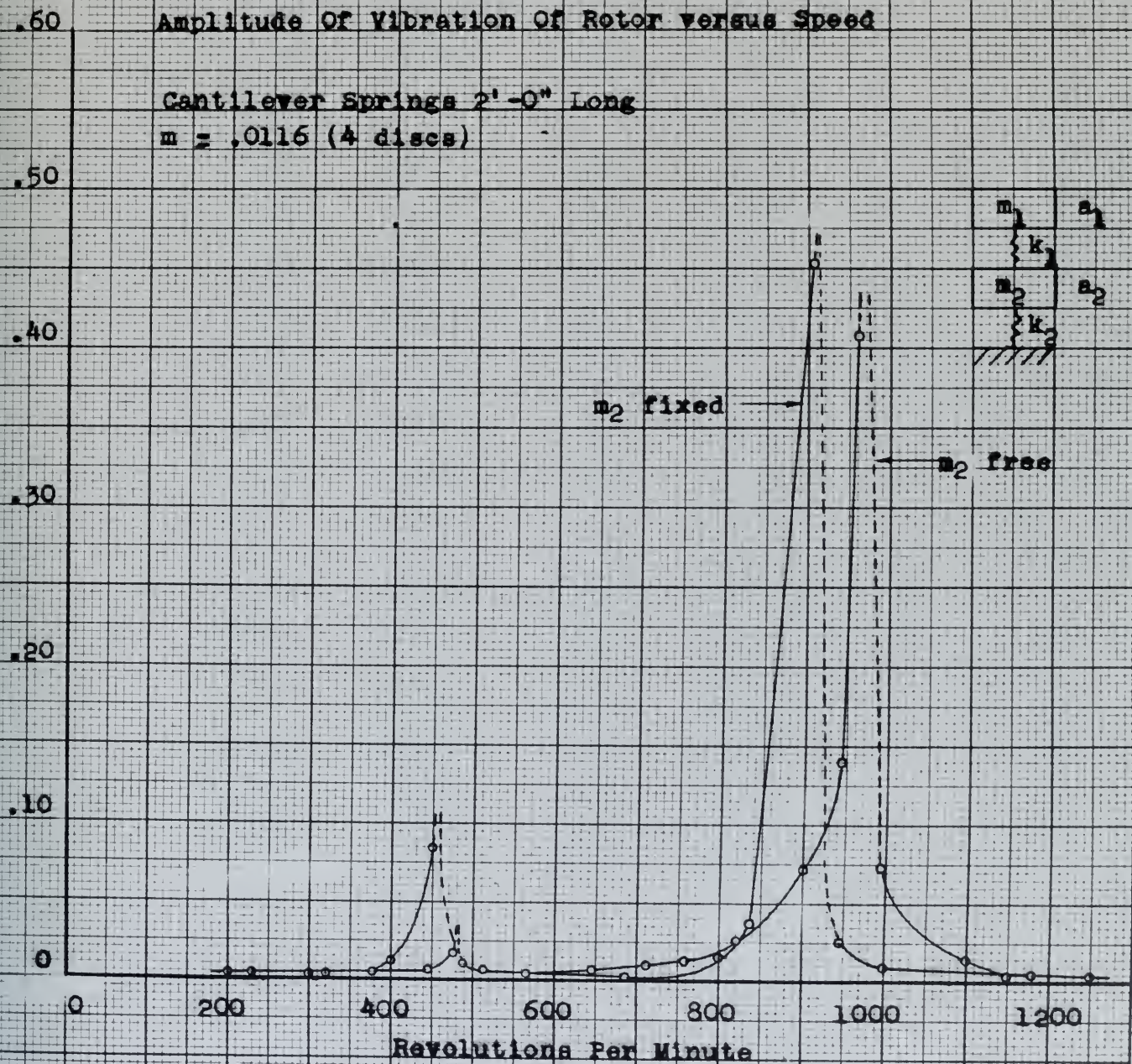
Cantilever Springs 2'-0" Long

 $m_1 = .0192$ (8 discs)

Amplitude a_1 (inches)

Amplitude Of Vibration Of Rotor versus Speed

Cantilever Springs 2'-0" Long

 $m = .0116$ (4 discs) m_2 fixed m_2 free

Revolutions Per Minute

DATA

Values of r.p.m. and corresponding amplitude of vibration of the rotor. Amplitudes are recorded in fractions of an inch.

Cantilever Springs 2'-0" Long

$$m_1 = .0192$$

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
200	.0036	200	.0036
315	.0048	250	.0036
380	.0060	370	.0060
410	.0190	400	.0140
420	.0330	445	.0600
440	.0150	465	.0140
520	.0060	520	.0096
600	.0060	630	.0072
670	.0060	730	.0120
710	.0400	800	.3550
740	.3130	850	.0048
770	.0036	950	.0024
1000	.0036	1000	.0024
1100	.0036	1120	.0024
1180	.0036	1190	.0024
1250	.0036	1250	.0024

Table

Table of the values of the function $f(x)$ for various values of x and y . The values are given in the following table.

Table of the values of the function $f(x)$ for various values of x and y .

$$f(x) = \frac{1}{2} \log \frac{1+x}{1-x}$$

x		y	
x	y	x	y
0.00	0.00	0.00	0.00
0.01	0.01	0.01	0.01
0.02	0.02	0.02	0.02
0.03	0.03	0.03	0.03
0.04	0.04	0.04	0.04
0.05	0.05	0.05	0.05
0.06	0.06	0.06	0.06
0.07	0.07	0.07	0.07
0.08	0.08	0.08	0.08
0.09	0.09	0.09	0.09
0.10	0.10	0.10	0.10
0.11	0.11	0.11	0.11
0.12	0.12	0.12	0.12
0.13	0.13	0.13	0.13
0.14	0.14	0.14	0.14
0.15	0.15	0.15	0.15
0.16	0.16	0.16	0.16
0.17	0.17	0.17	0.17
0.18	0.18	0.18	0.18
0.19	0.19	0.19	0.19
0.20	0.20	0.20	0.20
0.21	0.21	0.21	0.21
0.22	0.22	0.22	0.22
0.23	0.23	0.23	0.23
0.24	0.24	0.24	0.24
0.25	0.25	0.25	0.25
0.26	0.26	0.26	0.26
0.27	0.27	0.27	0.27
0.28	0.28	0.28	0.28
0.29	0.29	0.29	0.29
0.30	0.30	0.30	0.30
0.31	0.31	0.31	0.31
0.32	0.32	0.32	0.32
0.33	0.33	0.33	0.33
0.34	0.34	0.34	0.34
0.35	0.35	0.35	0.35
0.36	0.36	0.36	0.36
0.37	0.37	0.37	0.37
0.38	0.38	0.38	0.38
0.39	0.39	0.39	0.39
0.40	0.40	0.40	0.40
0.41	0.41	0.41	0.41
0.42	0.42	0.42	0.42
0.43	0.43	0.43	0.43
0.44	0.44	0.44	0.44
0.45	0.45	0.45	0.45
0.46	0.46	0.46	0.46
0.47	0.47	0.47	0.47
0.48	0.48	0.48	0.48
0.49	0.49	0.49	0.49
0.50	0.50	0.50	0.50

Cantilever Springs 2'-0" Long

$$m_1 = .0116$$

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
200	.0024	230	.0024
320	.0024	300	.0048
400	.0060	400	.0120
445	.0060	450	.0840
475	.0180	485	.0072
485	.0120	510	.0072
530	.0048	560	.0072
600	.0012	645	.0072
675	.0012	710	.0120
750	.0012	755	.0132
810	.0150	800	.0156
835	.0360	8200	.0300
905	.4530	900	.0720
945	.0240	945	.1400
1000	.0096	960	.4070
1150	.0036	995	.0720
1200	.0036	1040	.0480
		1100	.0120
		1170	.0024

3110. 2000

Cantilever Springs 1"-7" Long

$$m_1 = .0192$$

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
180	.0024	200	.0036
250	.0036	300	.0048
300	.0036	375	.0072
395	.0152	455	.0096
410	.0240	520	.0192
420	.0360	550	.0360
445	.0072	565	.1200
500	.0060	605	.0320
620	.0048	640	.0054
675	.0120	680	.0072
700	.0240	710	.0240
740	.3950	740	.0360
770	.0720	795	.1860
800	.0240	820	.4620
850	.0096	930	.0660
900	.0048	975	.0240
950	.0048	1035	.0180
990	.0048	1100	.0072
1030	.0048	1150	.0048
1120	.0036	1200	.0048

[illegible]

Cantilever Springs 1'-7"

$$m_1 = .0116$$

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
165	.0024	190	.0024
215	.0036	300	.0024
325	.0036	350	.0036
435	.0054	470	.0036
470	.0240	530	.0052
480	.0120	560	.0048
510	.0048	585	.0340
585	.0048	600	.0048
650	.0048	690	.0048
700	.0084	775	.0048
740	.0120	820	.0084
780	.0144	880	.0144
835	.0240	900	.0480
860	.0480	970	.3230
900	.3300	1010	.0720
930	.0820	1060	.0180
970	.0240	1100	.0120
1015	.0120	1160	.0096
1080	.0054	1210	.0048
1180	.0036		

Class Name		Class Name	
Symbol	Value	Symbol	Value
1000	1000	1000	1000
1001	1001	1001	1001
1002	1002	1002	1002
1003	1003	1003	1003
1004	1004	1004	1004
1005	1005	1005	1005
1006	1006	1006	1006
1007	1007	1007	1007
1008	1008	1008	1008
1009	1009	1009	1009
1010	1010	1010	1010
1011	1011	1011	1011
1012	1012	1012	1012
1013	1013	1013	1013
1014	1014	1014	1014
1015	1015	1015	1015
1016	1016	1016	1016
1017	1017	1017	1017
1018	1018	1018	1018
1019	1019	1019	1019
1020	1020	1020	1020
1021	1021	1021	1021
1022	1022	1022	1022
1023	1023	1023	1023
1024	1024	1024	1024
1025	1025	1025	1025
1026	1026	1026	1026
1027	1027	1027	1027
1028	1028	1028	1028
1029	1029	1029	1029
1030	1030	1030	1030
1031	1031	1031	1031
1032	1032	1032	1032
1033	1033	1033	1033
1034	1034	1034	1034
1035	1035	1035	1035
1036	1036	1036	1036
1037	1037	1037	1037
1038	1038	1038	1038
1039	1039	1039	1039
1040	1040	1040	1040
1041	1041	1041	1041
1042	1042	1042	1042
1043	1043	1043	1043
1044	1044	1044	1044
1045	1045	1045	1045
1046	1046	1046	1046
1047	1047	1047	1047
1048	1048	1048	1048
1049	1049	1049	1049
1050	1050	1050	1050
1051	1051	1051	1051
1052	1052	1052	1052
1053	1053	1053	1053
1054	1054	1054	1054
1055	1055	1055	1055
1056	1056	1056	1056
1057	1057	1057	1057
1058	1058	1058	1058
1059	1059	1059	1059
1060	1060	1060	1060
1061	1061	1061	1061
1062	1062	1062	1062
1063	1063	1063	1063
1064	1064	1064	1064
1065	1065	1065	1065
1066	1066	1066	1066
1067	1067	1067	1067
1068	1068	1068	1068
1069	1069	1069	1069
1070	1070	1070	1070
1071	1071	1071	1071
1072	1072	1072	1072
1073	1073	1073	1073
1074	1074	1074	1074
1075	1075	1075	1075
1076	1076	1076	1076
1077	1077	1077	1077
1078	1078	1078	1078
1079	1079	1079	1079
1080	1080	1080	1080
1081	1081	1081	1081
1082	1082	1082	1082
1083	1083	1083	1083
1084	1084	1084	1084
1085	1085	1085	1085
1086	1086	1086	1086
1087	1087	1087	1087
1088	1088	1088	1088
1089	1089	1089	1089
1090	1090	1090	1090
1091	1091	1091	1091
1092	1092	1092	1092
1093	1093	1093	1093
1094	1094	1094	1094
1095	1095	1095	1095
1096	1096	1096	1096
1097	1097	1097	1097
1098	1098	1098	1098
1099	1099	1099	1099

DATA

Values of γ found experimentally.

(γ = dimensionless coefficient of damping)

Cantilever springs	Value of γ
1 foot 7 inches long	.0915
2 feet 0 inches long	.0633

Rotor

$m_1 = .0116$.0422
$m_1 = .0192$.0294

TABLE

Values of Y from experimentally

(Y = dimensionless coefficient of resistance)

Centrifugal speed

1 foot 7 inches long

2 foot 11 inches long

3 foot 11 inches long

4 foot 11 inches long

5 foot 11 inches long

6 foot 11 inches long

7 foot 11 inches long

8 foot 11 inches long

9 foot 11 inches long

10 foot 11 inches long

11 foot 11 inches long

12 foot 11 inches long

13 foot 11 inches long

14 foot 11 inches long

15 foot 11 inches long

16 foot 11 inches long

17 foot 11 inches long

18 foot 11 inches long

19 foot 11 inches long

20 foot 11 inches long

21 foot 11 inches long

22 foot 11 inches long

23 foot 11 inches long

24 foot 11 inches long

25 foot 11 inches long

26 foot 11 inches long

27 foot 11 inches long

REFERENCES

1. Kimball, A.L. Vibration Prevention In Engineering.
John Wiley and Sons. 1932.
2. Timoshenko, S. Vibration Problems In Engineering.
Second Edition. D. Van Nostrand Co. 1937.
3. Stodola, A. Steam And Gas Turbines.
Volumes 1 and 2. McGraw-Hill Book Co. 1927.
4. Den Hartog, J.P. Mechanical Vibrations.
McGraw-Hill Book Co. 1934.
5. Hort, W. Technische Schwingungslehre.
Julius Springer.
6. Rodgers, C. On The Vibration And Critical Speeds Of Rotors.
Philosophical Magazine. Volume 44. July 1922.

Thesis

6318

M2

Mackay

The effect of flexible mountings upon the resonant speeds of machines having unbalanced motors.

Thesis

6318

M2

MacKay

The effect of flexible mountings upon the resonant speeds of machines having unbalanced motors.

thesM2

The effect of flexible mountings upon th



3 2768 001 88201 2

DUDLEY KNOX LIBRARY